considering several symmetric structures in systems with a regular disperse phase distribution [4].

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## COOLING OF A MAGNETIZED PLASMA AT A BOUNDARY WITH AN EXPLODING METAL WALL

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Cooling of a magnetized plasma at the boundary with a cold wall, which is accompanied by reaction of magnetic and thermal processes, leads in a number of cases to anomalously high effective thermal conductivity and magnetic diffusion coefficients. With cooling of a hydrogen plasma at a boundary with an insulator or a dense multicharged plasma, the effective thermal conductivity appears to be of the order of Bohm thermal conductivity [1, 2].

With cooling of a plasma bounded by a rigid and ideally conducting wall, as was shown in [1], the increase in thermal conductivity compared with classical magnetized thermal conductivity is less marked and it is only possible for a plasma with  $\beta \gg 1$  ( $\beta = 16\pi N_0 T_0/H_0^2$  is the ratio of thermal pressure of the plasma to magnetic pressure;  $N_0$ ,  $T_0$ , and  $H_0$  are density of electrons, temperature, and magnetic field in the plasma at a distance from the boundary). A metal wall may be considered to be rigid and ideally-conducting in the case when it does not explode due to action of heat flow from the plasma, i.e., its thermal conductivity in the condensed phase appears to be sufficient in order to remove heat without evaporating. This condition is fulfilled with relatively high energy densities (for plasma with  $T_0 = 1$ keV and  $\beta = 1$ , with  $H_0 \leq 0.2$  MG). With higher energy densities presence of an explosive heat flow for the metal markedly changes the nature of cooling and it increases heat losses for the plasma. This case is considered in the present work. However, the magnetic fields are not considered to be very high ( $H_0 > 10$  MG) since with  $H_0 < 10$  MG when there is explosion of a skin layer by Joule heat and the metal losses conductivity, the problem is reduced to that considered previously [1, 2] of plasma cooling at a boundary with an insulator.

Let all of the values depend on coordinate X perpendicular to the metal surface, and time t, magnetic field H, and electric field E are perpendicular to each other and axis X, and characteristic times are large compared with gas dynamic times, so that total pressure both in the hydrogen plasma and in metal vapor have time to equalize:

$$p + H^2/8\pi = P_0 \equiv 2N_0 T_0 + H_0^2/8\pi \tag{0.1}$$

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(p is thermal pressure). Equations for the magnetic and electric fields and the thermal balance for the plasma [3] are written in Lagrangian variables, and have the form

$$\frac{\partial E}{\partial X} = -\frac{1}{c} \left( \frac{dH}{dt} - \frac{H}{\rho} \frac{d\rho}{dt} \right), \quad \frac{\partial H}{\partial X} = -\frac{4\pi}{c} j, \qquad E = j/\sigma - \frac{b_A}{e} \frac{\partial T}{\partial X},$$

$$\rho \frac{de}{dt} - \frac{p}{\rho} \frac{d\rho}{dt} = -\frac{\partial Q}{\partial X} + jE, \quad Q = -\chi \frac{\partial T}{\partial X} + \frac{b_A T}{e} j,$$
(0.2)

where  $\rho$  is density of the mass;  $\varepsilon$  is internal energy;  $\sigma$ ,  $\chi$ ,  $b_{\Lambda}$  are transverse conduction, themal conductivity, and thermoelectric coefficients; Q is density of heat flow.

<u>1. Cooling of a Dense Plasma.</u> As shown in [1], existence of anomalously high effective thermal conductivity coefficients means that the problem for a hydrogen plasma is quasistationary: hydrogen plasma density in the boundary zone is large compared with density  $N_0$ , and

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in Eqs. (0.2) for magnetic and electric fields and the thermal balance for plasma in the boundary zone it is possible to ignore derivatives with respect to time and to assume the electric and energy flow are constant. Then (0.2) takes the form [2]

$$E = -\frac{c}{4\pi\sigma}\frac{\partial H}{\partial X} - \frac{b_{\Lambda}}{e}\frac{\partial T}{\partial X},$$

$$\frac{c}{4\pi}EH_{0}(1+5\beta/4) = -\chi\frac{\partial T}{\partial X} - \frac{cT}{4\pi e}b_{\Lambda}\frac{\partial H}{\partial X} + \frac{c}{4\pi}EH$$
(1.1)

 $[(c/4\pi)EH_0(1 + 5\beta/4)]$  is flow of energy into the discharge zone of the plasma].

Radiation plays the main role in heat transfer in ionized vapors. As a result of radiant thermal conductivity the mass of vapors in the discharge zone markedly exceeds the mass of the plasma. Therefore, the temperature of vapors is much lower than the temperature of the hydrogen plasma and the temperature of the hydrogen plasma at the boundary with the vapors may be assumed to be zero.

Thus, if the magnetic field at the boundary of a hydrogen plasma and vapors  $H_1$  is known, then boundary conditions for (1.1) will be

$$T(0) = 0, \ H(0) = H_1, \ N(-X_0) = 0 \tag{1.2}$$

(for the vapor boundary we assume that X = 0, for the discharge zone boundary  $X = -X_0$ ), and for the plasma the problem is reduced to one of cooling at a boundary with an insulator [2].

Since the main contribution to the mass of the plasma accumulated in the discharge gives a region in which the degree of electron magnetization  $\omega_e \tau_e \sim 1$ , and the total pressure  $P_0$  is prescribed, then for a changeover in (1.1) to dimensionless values as a unit for measuring pressure it is natural to take  $P_0$ , and units for measuring temperature [T] and density [N] so that with T = [T], N = [N] there is  $p = P_0$ , and  $\omega_e \tau_e \sim 1$ .

By selecting  $[T] = (ce\lambda \sqrt{mP_0})^{2/5}$ ,  $[N] = P_0/[T]$  (m is mass of an electron,  $\lambda$  is Coulomb logarithm) and introducing dimensionless coordinate  $x = -\frac{E}{e^{0,2}m^{0,2}c^{0,4}\lambda^{0,4}P_0^{0,2}}X$  and dimensionless  $\theta(x) = T/[T]$ , n(x) = N/[N],  $h = H/\sqrt{8\pi P_0}$ , the set of Eqs. (0.1), (1.1) may be rewritten in the form

$$2n\theta + h^{2} = 1, \quad \frac{4}{3} \frac{\alpha}{\theta^{3/2}} h' + b\theta' = 1,$$
  

$$b\theta h' + \frac{3}{4} \left( g + \frac{g_{1}}{\sqrt{912A}} \right) \theta^{5/2} \theta' = \frac{1 + 5\beta/4}{\sqrt{1 + \beta}} - h,$$
(1.3)

where  $\alpha$ , b, g, g<sub>1</sub> depend on the degree of magnetization  $y = \omega_e \tau_e = \frac{3}{2} \frac{\hbar}{n} \theta^{3/2}$  and are determined for example by approximation equations [3]

$$\alpha = 1 - \frac{6.42y^2 + 1.86}{\Delta}, \ b = \frac{y(1.5y^2 + 3.05)}{\Delta},$$
  

$$g = \frac{4.66y^2 + 12.1}{\Delta}, \ g_i = \frac{2y_i^2 + 2.64}{y_i^4 + 2.7y_i^2 + 0.677},$$
  

$$\Delta = 3.77 + 14.8y^2 + y^4, \ y_i = y/\sqrt{912A}$$
(1.4)

(A is atomic weight). Somewhat more accurate equations for  $\alpha$ , b, and g are given in [4]:

$$\alpha = 1 - \frac{3.03y + 1.37}{y^2 + 6.72y + 2.77},$$

$$= \frac{y(1.5y^2 + 2.54)}{y^3 + 7.09y^2 + 3.27y + 2.87}, \quad g = \frac{4.66y + 6.18}{y^3 + 5.35y^2 + 2.31y + 1.93}.$$
(1.5)

Boundary conditions (1.2) in dimensionless variables are written as  $h(0) = h_1 \equiv H_1 / \sqrt{8\pi P_0}$ , 0,  $n(x_0) = 0$ .

b

The mass of plasma accumulated in the boundary layer (mass of deposited plasma)  $a = \int_{0}^{x} \times \mathbf{N} \, dX$  will be determined in terms of dimensionless variables

$$a = \frac{P_0}{eE} \xi, \tag{1.6}$$

where

$$\xi = \int_{0}^{x_0} n dx. \tag{1.7}$$

since in view of the condition for freezing-in the magnetic field in a plasma at a distance from the discharge zone the plasma velocity is  $cE/H_0$ , then the rate of plasma mass accumulation

$$da/dt = N_0 c E/H_0. \tag{1.8}$$

By expressing E in terms of a using (1.6) we obtain a differential equation

$$a\frac{da}{dt} = \xi(\beta, h_1)\frac{c}{e}\frac{P_0N_0}{H_0}.$$

The value of  $\xi$  as a function of  $\beta$  and  $h_1$  was computed in [2] for kinetic coefficients (1.4).

For a complete statement of the problem it is necessary to determine magnetic field  $H_1$ . With quite high plasma density this may be done by means of a set of normal differential equations. In fact, the discharge resistance for a hydrogen plasma according to (1.6)-(1.8) falls with an increase in density  $N_0$ , but the discharge resistance for metal vapor does not depend on  $N_0$ ; therefore, with quite a high density a magnetized hydrogen plasma vapor conductivity may be ignored and it may be assumed that the magnetic field in them is constant  $(H = H_1)$ . In order to calculate it we use the condition for magnetic flux conservation

$$\frac{d}{dt}(H_1X_1) = cE \tag{1.9}$$

for a layer of vapors with thickness  $H_1$  and a condition for energy conservation assuming the vapors are an ideal gas with an adiabatic index  $\gamma$ :

$$\frac{d}{dt}\left(\frac{p_1X_1}{\gamma-1}\right) + p_1\frac{dX_1}{dt} + \frac{d}{dt}\left(\frac{H_1^2}{8\pi}X_1\right) + \frac{H_1^2}{8\pi}\frac{dX_1}{dt} = \frac{c}{4\pi}EH_0\left(1 + \frac{5}{4}\beta\right)$$
(1.10)

 $[p_1$  is thermal vapor pressure which is constant due to constancy of  $H = H_1$  and equilibrium condition (0.1)]. It should be noted that a condition for energy conservation is only used in the case when the geometry of the system is such that the flow of energy emitted from the vapor surface is completely balanced by that arising from the direction of the surrounding walls (closed system). In favor of the opposite limiting case when the flow of radiation moving away from the wall is not balanced (open system) is that given below. The set of Eqs. (1.6), (1.8)-(1.10), together with equilibrium conditions  $p_1 + H_1^2/8\pi = P_0$ , entirely determine cooling of a dense plasma.

With  $P_0$  = const,  $N_0$  = const,  $H_0$  = const,  $\beta$  = const we find that plasma deposition occurs by a diffusion rule

$$a = \sqrt{2\xi \frac{c}{e} \frac{P_0 N_0}{H_0}} t, \ E = \sqrt{\frac{\xi}{2ec} \frac{P_0 H_0}{N_0 t}},$$
  
$$X_1 = \sqrt{\frac{2\xi c}{e} \frac{P_0 H_0 t}{N_0 H_1^2}},$$
 (1.11)

and dimensionless magnetic field  $h_1$  is determined from an algebraic equation

$$h_1^2 \frac{\gamma - 2}{\gamma - 1} - h_1 \frac{2 + 2.5\beta}{\sqrt{1 + \beta}} + \frac{\gamma}{\gamma - 1} = 0,$$

i.e.,

$$h_{1}(\beta) = \frac{1}{(2-\gamma)\sqrt{1+\beta}} \left\{ \sqrt{1+\beta \left[1+\left(\frac{25}{16}\beta+\frac{3}{2}\right)(\gamma-1)^{2}\right]} - (\gamma-1)(1+5\beta/4) \right\}.$$
 (1.12)

With  $\beta \to 0$ ,  $h_1 \to 1$ , and with  $\beta \to \infty$ ,  $h_1 \simeq \frac{0.4\gamma}{(\gamma-1)\sqrt{\beta}} \to 0$ ; thus, according to [2] with  $\beta \to 0$ ,  $\xi \to 0$ 

and with  $\beta \to \infty$ ,  $\xi \to \text{const.}$  The relationship  $\xi(\beta, h_1(\beta))$  for  $h_1(\beta)$  from (1.12) obtained by means of Eqs. (1.3) and (1.7) for set of coefficients  $\alpha$ ,  $\beta$ , g from (1.5),  $g_1$  (A = 2) from (1.4) and with  $\gamma = 1.21^*$  is shown in Fig. 1 (curve 1). Given here for comparison is the relationship  $\xi(\beta, h_1(\beta))$  for set of coefficients (1.4) (curve 2). It is noted that although  $\frac{1}{2}\chi = 1.21$  corresponds approximately to the adjustic index for copper vapor in the region of

 $*\gamma = 1.21$  corresponds approximately to the adiabatic index for copper vapor in the region of mega-Gauss magnetic fields (see the next section).



the difference between coefficients (1.4) and (1.5) is quite marked (e.g., for b it reaches 30%), the difference between values of  $\xi(\beta)$  does not exceed 6%. This points to a weak dependence of  $\xi$  on the value of kinetic coefficients, and apparently in addition mutual balancing of deviations of a different sign for (1.4) from (1.5) for different  $\omega_e \tau_e$  is marked.

Since with large  $\beta$ ,  $h_1 \simeq 0$ , the effective diffusion coefficient  $D \sim 2\xi cP_0/eN_0H_0$  with cooling of a dense plasma at a boundary with an exploding metal wall is the same as with cooling at a boundary with an insulator  $[\xi(\beta \rightarrow \infty) \simeq 0.18$ ,  $D \simeq cT_0/eH_0]$  and it exceeds Bohm thermal conductivity by about an order of magnitude, but with  $\beta \simeq 1$  when  $\xi \simeq 0.3$  it appears to be of the order of Bohm thermal conductivity.

In order to check the theoretical results presented here determining cooling of a dense plasma close to an exploding metal wall, numerical modeling was performed for cooling of a deuterium plasma with  $T_0 = 0.5$  keV,  $H_0 = 0.5$  MG,  $\beta = 1$  close to a copper wall. In modeling, consideration was given (both for deuterium and for copper) to hydrodynamic movement, magnetic diffusion, and electron thermal conductivity. In addition, for deuterium the additional contribution was considered to the electric field and heat flow due to the Nernst effect [terms with coefficients  $b_{\Lambda}$  in (0.2)] and ionic thermal conductivity, and for copper this included radiant heat transfer. Kinetic coefficients  $\alpha$ ,  $b_{\Lambda}$ , g in deuterium were found from Eq. (1.5), and  $g_i$  from (1.4).

The equation of state for copper used in the calculations (in cm, g,  $\mu sec$ , temperature in eV) had the form

$$\begin{aligned} \varepsilon(\rho, \ T) &= \varepsilon_1(\rho) + \varepsilon_2(\rho, \ T) + \varepsilon_s(\rho, \ T), \\ p(\rho, \ T) &= p_1(\rho) + p_2(\rho, \ T) + p_s(\rho, \ T), \end{aligned}$$

where  $\varepsilon_1 = (2.32/\rho_0)(\delta^{2.1}/2.1 - \delta^{1.5}/1.5 + 4/21), \quad p_1 = 2.32(\delta^{3.1} - \delta^{2.5}) \quad (\rho_0 = 8.9 \text{ g/cm}^3, \quad \delta = \rho/\rho_0); \quad \varepsilon_2 = 0.0121 \ T^{3/4}\delta^{5/6}, \quad p_2 = (10/3)\varepsilon_2\rho; \quad \varepsilon_s = (0.965/A). [1.5(1 + z)T + Q(z)], \quad p_s = (0.965/A)\rho(1 + z)T; \text{ A is atomic weight equal to } 63.5; z \text{ was determined by an approximation method for solving the Saha equation for repeated ionization [5] using transcendental equation <math>I(z + 0.5) = T \ln(317A \ T^{3/2}/(z\rho));$ 

I(z) are ionization potentials; Q(z) are losses in ionization;  $Q(z) = \sum_{1}^{\infty} I(z)$ . Conductivity  $\sigma$  for copper was calculated in the plasma region ( $\rho < 0.28$  g/cm<sup>3</sup>) by equations in [6]:

for z > 1

$$\sigma = 0.871 \cdot 10^8 \frac{3.25 + 1.41/z}{1 + 1.41/z} \frac{T^{3/2}}{z\lambda},$$
$$\lambda = \ln\left(1 + \frac{0.052}{z} \sqrt{\frac{T^3A}{\rho z (1+z)}}\right);$$

for z < 1

$$\sigma = \frac{1}{0.594 \cdot 10^{-8} \frac{\lambda}{T^{3/2}} + \frac{1-z}{z} 1.3 \cdot 10^{-9}},$$

$$\lambda = \ln\left(1 + 0.037 \sqrt{\frac{T^3 A}{\rho z}}\right);$$
(1.13)

in the condensed phase region ( $\rho > 2.8 \text{ g/cm}^3$ )

$$\sigma = \frac{4.83 \cdot 10^8}{\varepsilon - 0.0004} \,\delta,\tag{1.14}$$

and in the intermediate region (0.28 g/cm<sup>3</sup> <  $\rho$  < 2.8 g/cm<sup>3</sup>)  $\sigma$  was determined by means of interpolation with respect to density between Eqs. (1.13) and (1.14). Electron thermal conductivity in copper was assumed to be unmagnetized, and the coefficient of electron thermal conductivity for copper was found from the Weideman-Franz rule

$$\chi = \frac{\pi^2 kT}{3e^2} \sigma$$

(k is Boltzman constant). For radiation energy transfer a forward-backward approximation [5] was used with path  $l = (4/3)l_r$  ( $l_r$  is Rosseland path) in order to provide correct limiting conversion to the thermal conductivity equation. The path depended on temperature and density (gray substance) and it was [5]

for 
$$z > 1$$
  $l_r = 0.3 \frac{T^{7/2}}{\rho^2} \frac{A^2}{z^3}$ ;  
for  $z < 1$   $l_r = 0.3 \frac{T^{7/2}}{\rho^2} \frac{A^2}{z^2}$ .

A boundary condition determining spreading of radiation was equality to zero of radiant flow at the boundary of the discharge zone (closed system). In the initial condition it is assumed that copper has a normal density  $\delta = 1$  and is close to room temperature,  $\varepsilon = 0.0013$ .

Presented in Fig. 2 are temperature and magnetic field profiles at instant of time t = 0.85  $\mu$ sec obtained by means of Eqs. (1.3) (solid curves) and as a result of numerical modeling (broken curves). Comparison points to satisfactory agreement. The agreement between values of cooled plasma volumes is much better. With numerical modeling the plasma thickness decreased at this instant of time by  $\Delta X = 0.042$  cm, and according to (1.11) for deposited plasma thickness  $\Delta X = 0.045$  cm.

2. Shunting Discharge for Metal Vapors. In the opposite case to that considered above, with quite low hydrogen plasma density the conductance of metal vapors is determinant, and the main role is played by the discharge for metal vapors thus shunting the discharge for the hydrogen plasma. We consider the problem in which in the initial instant of time a plasma with constant temperature, density, and magnetic field throughout the volume is in contact with a cold copper wall. Radiation, whose transfer with long times for closed system is found from the thermal conductivity equation, is fundamental in the heat transfer in metal vapors. Since a typical scale of length is absent from the problem, its solution is self-modeling and conductivity and magnetic diffusion are governed by the diffusion nature of self-modeling. Consideration of this problem differs from that of diffusion of a magnetic field accompanied by radiant thermal conductivity at an insulator [7] only in boundary conditions.

For the equation of state, radiation path, and conductance of copper vapors we assume a stepwise form for the dependences on temperature and density. Then by using for these values equations of the previous section in the regions of temperature 3-30 eV and density  $10^{-1}-10^{-3}$  g/cm<sup>3</sup>, we obtain approximately the following dependences:  $p/\rho = 0.0075T^{1.67}/\rho^{0.14}$ ,  $\ell_{\rm T} = 10^{-6}T/\rho^{1.64}$ ,  $\sigma = 2.7 \cdot 10^8 T^{0.92} \rho^{0.2}$ , adiabatic index  $\gamma = p/\epsilon\rho + 1 = 1.21$ . In order to change over to dimensionless variables, units for measuring temperature [T] and density  $[\rho]$  are chosen similar to [7] so that thermal diffusion and magnetic diffusion coefficients  $\varkappa = c^2/4\pi\sigma$  are a single order of value:  $\sigma_{SB} [T]^4 l_r([T], [\rho])/P_0 = \varkappa([T], [\rho])$  ( $\sigma_{SB} = 1.03 \cdot 10^{-6}$  is Stefan-Boltzman constant), and thermal pressure is of the order of the prescribed  $P_0$ :  $p([T], [\rho]) = P_0$ ; then  $[T] = 12 \text{ eV } P_0^{0.31}$  (GPa),  $[\rho] = 0.01 \text{ g/cm}^3 P_0^{0.57}$ (GPa). By using the self-modeling variable  $\zeta = 390 \int \rho dX$   $(g/cm^2)/[\sqrt{t(\mu sec)}P_0^{0.37}$ (GPa)] and introducing dimensionless functions

$$T = [T]\theta(\zeta), \ \rho = [\rho]n(\zeta), \ H = \sqrt{8\pi P_0}h(\zeta),$$
  

$$E = 0.63 \frac{\text{kv}}{\text{cm}} \frac{P_0^{0,3} \text{ (GPa)}}{\sqrt{t \text{ (} \text{µsec}\text{)}}} \varepsilon(\zeta), \ Q = 2.5 \cdot 10^8 \frac{\text{W}}{\text{cm}^2} \frac{P_0^{0,3} \text{ (GPa)}}{\sqrt{t \text{ (} \text{µsec}\text{)}}} q(\zeta),$$
  

$$X = 0.25 \text{ cm} \frac{\sqrt{t \text{ (} \text{µsed}\text{)}}}{P_0^{0,2} \text{ (} \text{GPa)}} x(\zeta),$$



set of Eqs. (0.1), (0.2) is rewritten in the form

$$\theta^{1,67} n^{0,86} + h^2 = 1,$$

$$\frac{d\varepsilon}{d\zeta} = \frac{\zeta}{n} \left( \frac{dh}{d\zeta} - \frac{h}{n} \frac{dn}{d\zeta} \right), \quad \frac{dh}{d\zeta} = -\frac{1}{2} \frac{\theta^{0,92}}{n^{0,8}} \varepsilon, \quad q = -\frac{16}{3} \frac{\theta^4}{n^{0,64}} \frac{d\theta}{d\zeta},$$

$$\frac{dq}{d\zeta} = \frac{\varepsilon^2}{2} \frac{\theta^{0,92}}{n^{0,8}} + \frac{5}{6} \zeta \left( \frac{14}{3} \frac{\theta^{0,67}}{n^{0,14}} \frac{d\theta}{d\zeta} - \frac{\theta^{1,67}}{n^{1,14}} \frac{dn}{d\zeta} \right), \quad \frac{dx}{d\zeta} = 1/n.$$

$$(2.1)$$

We formulate boundary conditions for system (2.1). Heat flow at the boundary with the plasma ( $\zeta = 0$ ) may be calculated as the difference between total energy flow arriving in the discharge zone of the plasma ( $c/4\pi$ )EH<sub>0</sub>(1 + 5 $\beta$ /4), and the flow of electromagnetic energy ( $c/4\pi$ )EH<sub>1</sub> (H<sub>1</sub> is magnetic field at the boundary of the hydrogen plasma and vapors). Thus, one of the boundary conditions at this boundary is the connection between heat flow and the electric field which in dimensionless variables appears as

$$q(0) = \left[\frac{1 + (5/4)\beta}{\sqrt{1+\beta}} - h_1\right]\varepsilon(0).$$
(2.2)

The second boundary condition emerges from the fact that the discharge for vapors is shunting for the hydrogen plasma discharge, i.e., for the hydrogen plasma discharge in this case we assume that E = 0, and this means from (1.11)  $\xi = 0$ , and since  $\zeta$  reverts to zero with  $h_1 = 1$ , then the boundary condition should be assumed as

$$h_1 = 1.$$
 (2.3)

At the boundary of vapors with unevaporated metal boundary conditions will equal zero for temperature, heat flow, and electric field:

$$\theta(\zeta_0) = q(\zeta_0) = \varepsilon(\zeta_0) = 0. \tag{2.4}$$

By solving set (2.1) with boundary conditions (2.2)-(2.4) using (1.8) we determine the thickness of deposited plasma:

$$\Delta X = 0.25 \text{ cm} \frac{\sqrt{i (\text{lisec})}}{P_0^{0,2} (\text{GPa})} \varepsilon(0) \sqrt{1+\beta}.$$
(2.5)

We estimate the order of values characterizing the discharge zone with large and small  $\beta$ . With  $\beta \gg 1$  in equations for heat transfer it is possible to ignore the role of terms with the magnetic field, the electric field is found from relationship (2.2), then  $\theta \sim \beta^{0,10}$ ,  $n \sim \beta^{-0,22}$ ,  $x \sim \beta^{0,47}$ ,  $\varepsilon(0) \sim \beta^{-0,03}$ , and the magnetic field fades exponentially into the depth of the discharge zone. With  $\beta \ll 1$  in the discharge zone  $\theta^{1,67}n^{0,86} \sim \beta$ , and from (2.1) we have

$$\theta \sim \beta^{0,42}, \ n \sim \beta^{0,35}, \ x \sim \varepsilon(0) \sim \beta^{0,27}.$$
 (2.6)

Some results of numerical calculations of set (2.1) with boundary conditions (2.2)-(2.4) are presented in Figs. 3 and 4. Shown in Fig. 3 as a function of  $\beta$  are the electric field at the entrance to the discharge  $\varepsilon(0)$ , the mass of vapors in the discharge  $\zeta_0$  magnetic field at the boundary with unevaporated metal  $h(\zeta_0)$ , and the ratio of volume occupied by vapors

 $X(\zeta_0)$  to the volume of deposited plasma  $s = x(\zeta_0)/\epsilon(0) \sqrt{1+\beta}$ . Solid curves in Fig. 4 are profiles for the magnetic field and temperature found as a result of solving Eq. (2.1) and converted for comparison with results of numerical modeling in dimensional units for the case of plasma cooling with  $H_0 = 1$  MG,  $T_0 = 10$  keV,  $\beta = 1$  at instant of time t = 0.035 µsec. Given here for this case (broken curves) are the results of numerical modeling obtained in the manner described in part 1. Numerical modeling gives a smaller size of the region occupied by the discharge which is explained by the effect of the hydrogen plasma discharge which is not considered in calculation by Eq. (2.1). This difference appears in the thickness of deposited plasma, which according to (2.5) is  $\Delta X = 0.047$  cm, and with numerical modeling is  $\Delta X = 0.036$  cm.

Until now we considered the system to be closed so that the flow of energy radiated from the surface of vapors is completely balanced by radiation arriving from the direction of the surrounding walls. We move to the case of an open system when the geometry is such that this flow is not balanced by anything. Then, in vapors due to release of heat, the characteristic thermal pressure is much less than the magnetic and thermal pressure of a hydrogen plasma  $\beta_V \ll 1$ , and  $\beta$  changes with time. In order to determine relationship  $\beta_V(t)$  it is considered that heat flow from the surface, corresponding in this case to black body radiation Q ~ T<sup>4</sup>(t), should be balanced by heat arriving from the plasma ~E(t) ~  $\epsilon(0)/\sqrt{t}$ . By equating these flows and using dependences for  $\theta$  and  $\epsilon(0)$  on  $\beta$  from (2.6), we obtain  $\beta_V \sim t^{-0.35}$ , X ~  $t^{0.4}$ . Thus X increases more slowly than by a diffusion rule, and the discharge resistance for vapors for an open system is less than for a closed system.

We consider the question of when a plasma may be considered to be quite dense and for cooling of it to use the results of part 1, and when to use part 2 for which, as already said, the resistance of a discharge for a hydrogen plasma and for metal vapors should be compared [by comparing the thickness of deposited plasma calculated by Eqs. (1.11) and (2.5)]. A governing factor will be the regime for which the thickness appears to be less, although if these values are not strongly different it is possible to expect a marked effect of the unconsidered regime (as for example for the case in Fig. 4) of decreasing deposited plasma thickness. However, in any of these regimes if magnetization of the plasma is quite high, the effective thermal conductivity may markedly exceed classical thermal conductivity.

## LITERATURE CITED

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